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CAUSALITY BETWEEN GOVERNMENT EXPENDITURE AND ECONOMIC GROWTH IN ALGERIA: EXPLOSIVE BEHAVIOR TESTS AND FREQUENCY DOMAIN SPECTRAL CAUSALITY

***Abstract.** This paper examines the causal relationship between government expenditure and economic growth in Algeria over the period of 1980-2017 using the six equations of Wagner's law, based on an econometric study using both the Supremum ADF (SADF) and generalized supremum ADF (GSADF) tests for explosive behaviors. We also employed the Johansen co-integration test and the frequency domain spectral causality depending on the Breitung-Candelon (2006) procedure. The objective of this paper is to determine which hypothesis held in the context of Algeria in the short and long-run terms, so we need to select among Wagner's law hypotheses and Keynesian view hypotheses. The findings of co-integration revealed the existence of long run relationship between the variables, while, the causality test has revealed a unidirectional causal relationship running from economic growth to government expenditure in three equations among six equations when we do not deal with the GDP and government expenditure per capita.*

***Keywords:** Government expenditure, Economic growth, Wagner's law, Explosive behaviors, Frequency domain spectral causality.*

JEL Classification: C02, C11, C45, C46, C63

1. Introduction

The relationship between government expenditure (education, health, defense, investment, research and development, subsidization, etc.) and economic growth, especially the causal relationship has been a big concern for the last two centuries, particularly after the Russian Revolution in 1917 and the Great Depression in 1929, Kolcak et al (2017) declared that the theory "Laissez-Faire" which was the prevailing economic thought at the end of the 19th century and the beginning of the 20th century was the main result behind the Great Depression (1929) and the huge decrease in the whole world public expenditures.

The main problematic in the relationship between government expenditure and economic growth is to define the endogenous variable and the exogenous variable. From previous literature, we found two major answers, Adolph Wagner (1893) was the first to stipulate the causal relationship between government expenditure and economic growth, based on Wagner's law, the causal relationship is running from economic growth to government expenditure, in other words, Wagner observed that when the real income per capita (real GDP per capita) increases, the size of government (the ratio of government expenditure to total output of an economy) also increases, subsequently, Wagner's law suggests that the economic growth is the key determinant of government expenditure, meaning that the economic growth represents the exogenous factor when the government expenditure is the endogenous one. In 1936, the British John Maynard Keynes, came with the Keynesian hypotheses, which postulates that the causal relationship is running from government expenditure to economic growth not the opposite, representing that the government expenditure is the exogenous indicator, which must be used to impulse and accelerate the economic growth, but it should be noted that Ram (1986) examined this relationship on 63 developed and developing countries and he found that there is no evidence of any causal relationship between government expenditure and economic growth, while Abu-Badr and Abu-Qarn (2003) conducted the same study on 3 MENA countries and they established a bidirectional causal relationship between the two variables. Therefore, from this brief overview, it seems that there are four possibilities of causal relationship between government expenditure and economic growth as follows:

- 1- Wagner's law hypotheses: unidirectional causal relationship running from economic growth to government expenditure;
- 2- Keynesian hypotheses: unidirectional causal relationship running from government expenditure to economic growth;
- 3- Feedback hypotheses: bidirectional causal relationship between government expenditure and economic growth;
- 4- Neutrality hypotheses: no causal relationship between government expenditure and economic growth.

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During the last decades, many scholars tried to examine the Wagner view using different methods and proxies for the government expenditure and the economic growth. And we can summarize these equations as follows:

- Equation 1: $RGE = f(RGDP)$ **Peacock-Wiseman (1961)**
Equation 2: $RGCE = f(RGDP)$ **Pryor (1968)**
Equation 3: $RGE = f(RGDP/N)$ **Goffman (1968)**
Equation 4: $RGE/N = f(RGDP/N)$ **Gupta (1967). Michas (1975)**
Equation 5: $RGE/RGDP = f(RGDP/N)$ **Musgrave (1969)**
Equation 6: $RGE/RGDP = f(RGDP)$ **Mann (1980)**

Where: RGE is the real government expenditure; RGDP is the real GDP; N is the population; RGCE is the real government consumption expenditure; RGDP/N is the real GDP per capita; RGE/N is the real government expenditure per capita and RGE/RGDP is the ratio of government expenditure to the real GDP.

Algeria is the 10th largest country in the world by 1381741 km², making it as the largest country in Africa and Arabic world, with a population estimated at 42 million in 2018. Also, this country is one of the most important oil exporters in the world by 1348361 barrels per day (2018 data) ranked 18th globally and 9th in OPEC, the Algerian oil and gas sector accounts for more than 25% of GDP and more than 97% of exports earnings. Therefore, the Algerian economy depends a lot on the oil prices fluctuations, so in this case; testing the relationship between government expenditure and economic growth in an oil exporter country is a very big concern for the policymakers, especially for Algeria and after the collapse of world oil prices beginning in 2015.

The government expenditure plays a crucial role in the Algerian economy with government size around 20% in 2017. Thus, the fiscal policy in Algeria is the most important macroeconomic policy instrument, historically; the total government expenditure increased by 52% between 1990 and 2017 and by 164% between 1980 and 2017, fig 1 shows that the first substantial increase in the government expenditure occurred in 1992 and between 2010 and 2012 according to the high oil prices in this period, the figure shows also the similarity behavior between the two curves which appear the same changes in the two variables especially in 2008, 2010, 2012, 2013 and 2015.

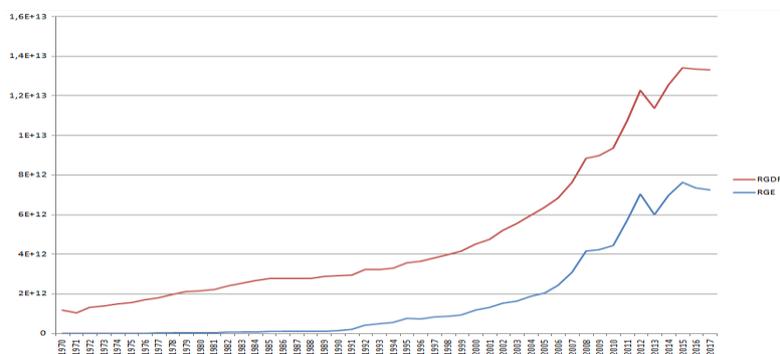


Figure 1. Government expenditure and real GDP in Algeria over the period 1970-2017 (real values in dollars based on World Bank database 2019)

Consequently, the target of this paper is to examine the causal relationship between government expenditure and economic growth in Algeria over the period 1970-2017. This investigation aims at making a contribution firstly in the literature review by examining the six equations of Wagner's law and making a comparison between the equations and the proxies of both government expenditure and economic growth. Secondly, the paper uses the modern SADF (supreme ADF) and GSADF (generalized sup ADF) tests to investigate whether the series have an explosive behavior which allows us to make a comparison between the series behavior. Thirdly, this study unlike the previous studies uses the frequency domain spectral causality by decomposing the total spectral interdependence between government expenditure and economic growth into short and long- run periods. The rest of this paper is organized as follows, section 2 deals with the literature review, section 3 presents the model, data and methodology of study, section 4 shows the results of the econometric study, when section 5 summarizes the paper by presenting the recommendations.

2. Literature review

There are many studies that have attempted to test the causal relationship between the government expenditure and the economic growth by using different proxies and several econometric approaches on different countries individually or as a group; therefore, many papers aim at determining the existence and the direction of the causality between the two variables and to confirm whatever there is an evidence of Wagner or Keynesian views (in addition the feedback and neutrality hypotheses), the Wagner's law is supported in several investigations as Abizadeh and Yousefi (1998) in Korea; Islam (2001) in USA; Burney (2002) in Kuwait; Al-Faris (2002) in Gulf countries; Halicioglu (2003) in Turkey; Kalam and Aziz (2009) in Bangladesh; Abdullah and Maamor (2009) in Malaysia; Satish and Rahul (2010) in India; Salih (2012) in Sudan; Ono (2014) in Japan; Cavicchioli and

Piostresi (2016) in Italy and many others. On the other hand, many scholars found the Keynesian hypothesis such as Dristakis and Adamopoulos (2004) in Greece; Loizedes and Vamvoukas (2005) in Ireland, Greece and UK; Liu et al (2008) in USA; Shahbaz et al. (2013) in Portugal; Chipaumire et al (2014) in South Africa; Ebaid and Bahari (2019) in Kuwait, in addition, some analyses like Sigh and Sahni (1984) and Abu Badr and Abu Qarn (2004) supported the feedback hypotheses in three MENA nations, and few investigations established the neutrality hypotheses as Ram (1986) in 63 developing and developed countries and Ele (2014) in Nigeria.

Therefore, from this brief background, we can say that there is a scarcity or a lack of studies in the case of Algerian economy, so we want to fill this gap and examine the existence and the direction of the causal relationship between the two variables in the case of Algeria.

3. Data and Methodology

3.1. Methodology of study

3.1.1. Testing for bubbles

Testing for bubbles is the series of the most modern tests in unit root tests specialized in the explosive roots, recently, Phillips, Wu and Yu (2011) or (PWY) and Phillips, Shi and Yu (2013) (PSY) based on recursive and rolling Augmented Dickey Fuller (ADF) unit root developed new strategies to detect bubbles in the series using right tail ADF with the null hypotheses of a unit root against the alternative hypotheses of a mildly explosive process, the PWY strategy according to Homm and Breitung (2012) is the best to detect the periodically collapsing bubbles and in real time monitoring. Also, Phillips et al. (2013) show that the PSY strategy is better than PWY to detect the presence of multiple bubbles.

Following the PSY strategy, we have the following random walk process with an asymptotically negligible drift based on the familiar ADF test:

$$\Delta y_t = \alpha + \gamma y_{t-1} + \sum_{i=1}^p \delta_i \Delta y_{t-i} + \varepsilon_t \quad (1)$$

Where y_t is the variable in testing; α is an intercept; P is the maximum lag and ε_t is the error term.

The two hypotheses in this case to test for an explosive behaviour (bubble) are the unit root (null hypotheses) against the mildly explosive autoregressive coefficient (alternative hypotheses), for this reason we have:

- The interval of test is $[0,1]$.
- δ_{r_1,r_2} and ADF_{r_1,r_2} are the coefficients estimated by equation (1) and its ADF statistic over the interval $[r_1,r_2]$.
- r_ω is the window size of the regression ($r_\omega = r_2 - r_1$) and r_0 is the fixed initial window.
- r_1 and r_2 are the starting points and ending points respectively.

In the standard ADF unit root test r_1 and r_2 are normally the first and last observations respectively which makes $r_\omega = r_0 = 1$ as in Fig.2.

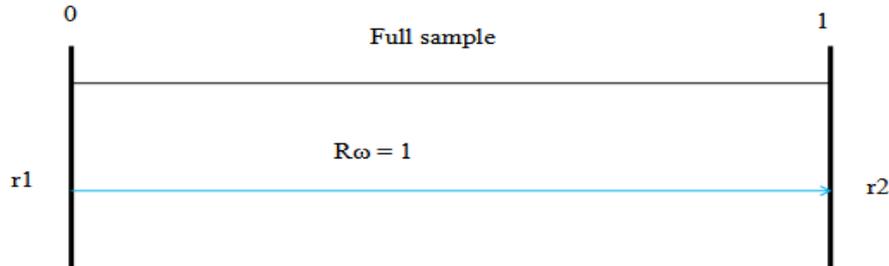


Figure 2. Illustration of the ADF procedure

The second test is the RADF test (Rolling ADF test) presented by Taipalus (2006) and Virtanen and Taipalus (2018), Fig.3 shows the procedure of the test when the test is a rolling version of ADF test which is calculated over a rolling window of fixed size ($r_\omega = r_0$), at each step in this test, the window starts and ends in r_1 and r_2 respectively with an ADF statistic for each window denoted as ADF_{r_1, r_2} , for this reason RADF statistic is the supremum ADF_{r_1, r_2} statistic for all possible windows (for example 3 windows in Fig.3).

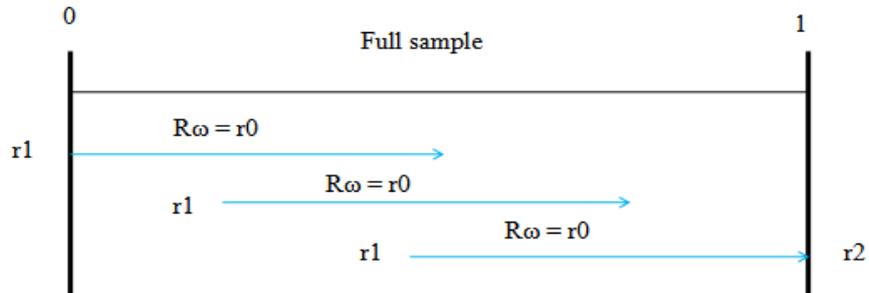


Figure 3. Illustration of the RADF procedure

The third test is suggested by PWY named the SADF test (Sup ADF test), the difference between SADF and RADF is the starting points r_1 which is fixed in SADF test as presented in Fig.4, and r_2 (end point) is set depending on the choice of minimal windows size, which makes r_0 is $r_\omega = r_2$, and each step yields an ADF statistic denoted as ADF_{r_2} , the SADF statistic is defined as the supremum ADF_{r_2} for $r_2 \in [r_0, 1]$.

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} \{ADF_{r_2}\} \quad (2)$$

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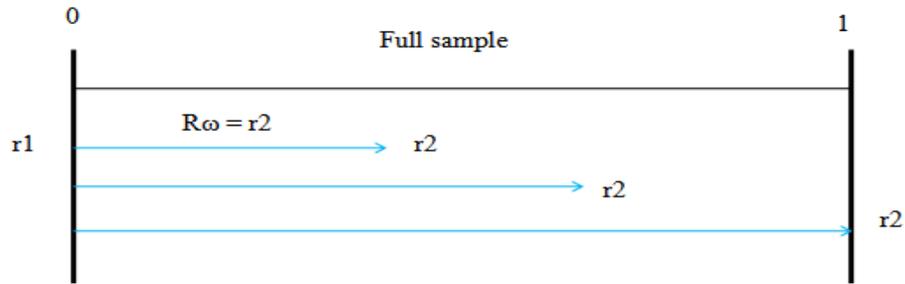


Figure 4. Illustration of the SADF procedure

The last test is presented by PSY named the Generalized SADF test (GSADF test) developed by Phillips et al. (2015), unlike the SADF procedure, GSADF procedure has various starting points (r_1) as in Fig.5 within the interval $[0, r_2 - r_1]$, and GSADF statistic is defined as the supremum $ADF_{r_1}^{r_2}$ for $r_1 \in [0, r_2 - r_1]$ and $r_2 \in [r_0, 1]$.

$$GSADF(r_0) = \sup_{r_2 \in [r_0, 1]; r_1 \in [0, r_2 - r_0]} \{ADF_{r_1}^{r_2}\} \quad (3)$$

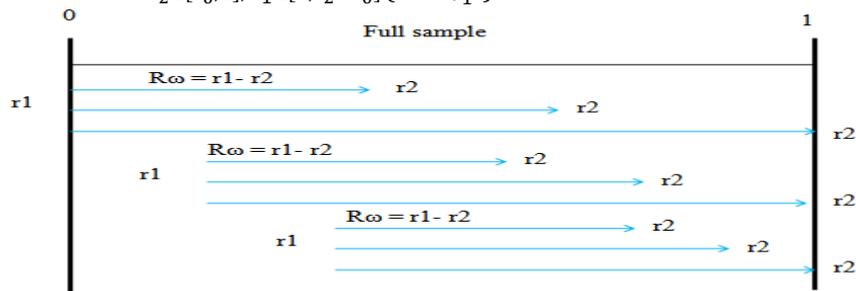


Figure 5. Illustration of the GSADF procedure

3.2.2. Frequency domain spectral causality

As mentioned by Pavia et al. (2008) the time domain (Granger causality and TYDL procedure) graph shows the signal change over the time, but the frequency domain graph shows how the signal lies within each given frequency (ω) band of the frequencies, Geweke (1982) showed that the causality among a bivariate series can be detected at a particular frequency by composing spectral density (Geweke, 1982 and Hosoya, 1991), Yao and Hosoya (2000) developed a Wald test methodology for causality at same given frequency based on nonlinear restrictions on the auto-regressive parameters, then, Breitung and Candelon (2006) used a bivariate VAR model and they proposed a new procedure based on a set of linear hypotheses on the AR parameters and it can be easily generalized to allow for co-integration relationships and higher dimensional systems.

According to Fritsche and Pierdzioch (2016) whom used the VMA (Vector Moving Average) of the bivariate VAR model as follows:

$$y_t = \psi(L)\varepsilon_t \quad (4)$$

Where ε_t is the white noise distribution; L is the lag operator and $\psi(L)$ is the lag polynomial.

Following vector shows the partitioning of $\psi(L)$ into parts as

$$\psi(L) = \begin{bmatrix} \psi_{11}(L) & \psi_{12}(L) \\ \psi_{21}(L) & \psi_{22}(L) \end{bmatrix} \quad (5)$$

In this case Geweke (1982) suggests to test the Granger non causality as a specific frequency ω the following measure $M_{y_1 \text{ cause } y_2}(\omega)$ which can be calculated as follows:

$$M_{y_1 \text{ cause } y_2}(\omega) = \log \left[1 + \frac{|\psi_{12}(e^{-i\omega})|}{|\psi_{11}(e^{-i\omega})|} \right] \quad (6)$$

Where i : is an imaginary number.

The next step is to test whether y_1 causes y_2 ($M_{y_1 \text{ cause } y_2}$) at any frequency ω , so we test the null hypotheses $H_0: M_{y_1 \text{ cause } y_2}(\omega) = 0$ (Geweke, 1982), Breitung and Candelon (2006) proposed a modified frequency domain causality using the VAR specification as follows:

$$M_t = \omega_1 M_{t-1} + \dots + \omega_p M_{t-p} + \dots + \partial_1 N_{t-1} + \partial_p N_{t-p} + \emptyset_t \quad (7)$$

And the new null hypotheses became $H_0: R(\omega)\Omega$ where Ω constitutes a vector of coefficients of N and

$$R(\omega) = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \dots & \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) & \dots & \sin(p\omega) \end{bmatrix} \quad (8)$$

The F statistic for this equation follows F (2, T-2p) for $\omega \in (0, \pi)$, and it is necessary to be noted that the high frequencies represented the short-run term causality and the low frequencies represented the long-run term causality, and as considered by Toda and Phillips (1993) in co-integration systems the definition of the causality of frequency zero is equivalent to the concept of long-run causality.

3.2. Data and Model of study

We employed several variables in our paper, the real GDP (RGDP); the real government expenditure (RGE); the real government consumption expenditure (RGCE); the real GDP per capita (RGDP/N); the real government expenditure per

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capita (RGE/N) and the ratio of government expenditure to real GDP (RGE/RGDP) for Algeria during the period of 1970-2017. The data are from the World Bank (2019) database and from the National Statistics Office of Algeria.

4. Results of study

4.1. Testing for bubbles (explosive behaviours)

We employed firstly the two methods of SADF and GSADF tests to examine the behavior of our series if there is an explosive behavior over the period of the study or not. Table one show the results with the critical values which are calculated by the Monte Carlo simulations with 1000 replications. The window size obtained for the two tests is 13, so it is clear that both of RGDP and RGE series have an explosive behavior (we focus just on these two series which are the most important to explain the government expenditure and the real GDP), according to Fig.6, the most important bubble in the GDP series was during the period 2001-2008, this bubble (explosive behavior) is followed by the black decade in Algeria (1990-2000) which characterized the insecurity in the country, during this period, the country experienced a significant economic deterioration in all sectors. Also, it is clear that the bubble finished in 2008 due to the financial crisis which happened in 2008, and we observed that there is a new small bubble over the period 2012 - 2014 due to the high oil prices in this period. In addition, the Fig.7 shows that the GSADF curve for the government expenditure has two major bubbles. The first one was during the period of 1990-1996 (the most insecurity period in Algeria after the independence in 1962). This bubble is due to the expenditure in the military sector, because the government tried to fight the terrorist groups with a huge military expenditure and subsidies to protect the Algerian citizens. However, the second bubble occurred during the period of 2002-2014, and this period can be called period of return to the path of development, where the government injected considerable funds into the economy through the five-year plans (2001-2004; 2005-2009; 2010-2014 and the last 2015-2019). Therefore, the unit root bubbles test determined that the two variables do not have the same behavior over the period of 1970-2017, especially after the black decade (2000-2017), figures 8 to 11 in the appendixes show the other date-stamping bubble periods curves for the 4 four other variables with the same results (same bubbles periods) as in RGDP and RGE series except the RGE/RGDP series where there is no bubbles.

Table 1. SADF and GSADF tests results

Variables	SADF		GSADF	
	Statistic	P-value	Statistic	P-value
RGDP	0.4682	0.0390**	1.5919	0.0090***
RGE	7.3597	0.0000***	7.3597	0.0000***
RGCE	0.7103	0.0190**	1.4085	0.0140**

RGDP/N	-0.5912	0.3650	1.2699	0.0220**
RGE/N	6.0655	0.0000***	6.0655	0.0000***
RGE/RGDP	-1.7742	0.9120	0.1915	0.3310
Critical values				
90%	0.8729		1.5787	
95%	0.3890		0.9844	
99%	0.0478		0.7189	
*** denotes significance at 10, 5 and 1% respectively.				

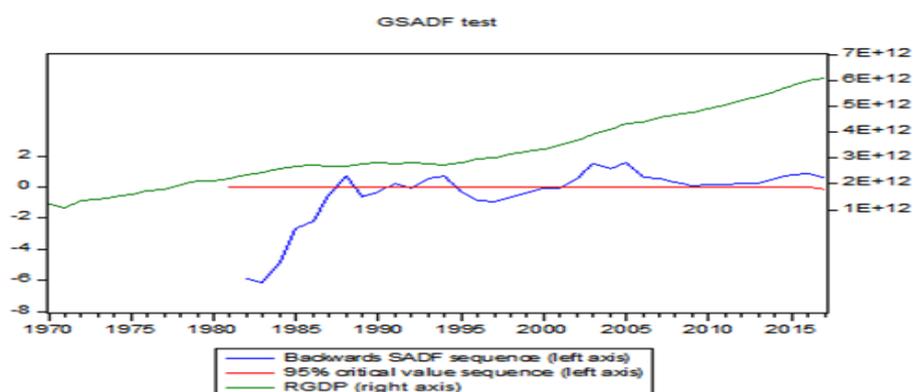


Figure 6. Date-stamping bubble periods in RGDP (GSADF TEST)

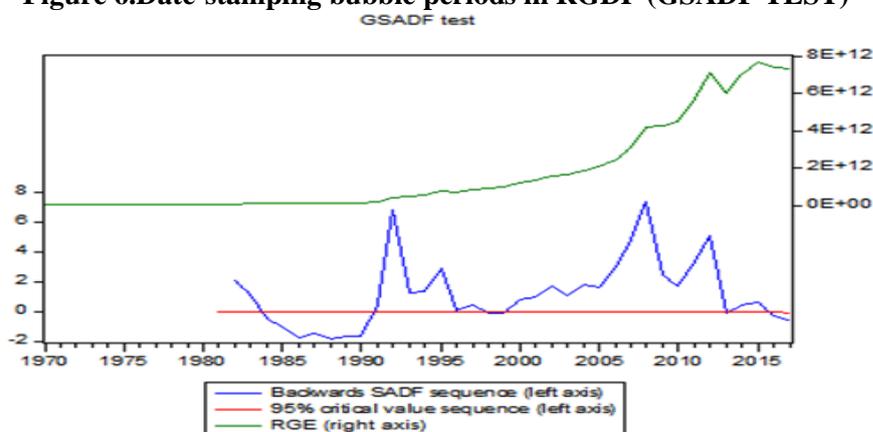


Figure 7. Date-stamping bubble periods in RGE (GSADF TEST)

4.2. Unit root test

As usual, the first step in time series analysis is the unit root test, for this reason, we applied the Phillips-Perron test (PP test) for the 3 equations, which are with trend and constant, with only constant and without any constant or trend. The results obtained from table 2 shows that all the variables are I (1) series, so we can

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apply the Johansen-Juselius (2002) co-integration for all the equations of Wagner's law as we discussed in the introduction.

Table 2. Unit root test results

Variables	with constant		with constant and trend		without constant and trend	
	Statistic	P-value	Statistic	P-value	Statistic	P-value
RGDP	1.867	0.999	-0.328	0.987	7.037	1.000
RGE	2.194	0.999	-0.521	0.979	3.502	0.999
RGCE	0.869	0.994	-0.971	0.938	4.478	1.000
RGDP/N	-0.838	0.798	-1.605	0.776	1.948	0.986
RGE/N	1.395	0.998	-1.111	0.916	2.698	0.997
RGE/RGDP	-3.055	0.038**	-3.438	0.058*	0.840	0.889
Δ .RGDP	-6.084	0.000***	-6.307	0.000***	-2.698	0.008***
Δ .RGE	-6.544	0.000***	-7.342	0.000***	-6.267	0.000***
Δ .RGCE	-5.473	0.000***	-5.548	0.000***	-3.825	0.000***
Δ .RGDP/N	-7.757	0.000***	-7.717	0.000***	-7.019	0.000***
Δ .RGE/N	-7.702	0.000***	-7.342	0.000***	-6.267	0.000***
Δ .RGE/RGDP	-7.557	0.000***	-7.492	0.000***	-7.491	0.000***

Δ .: denotes the first differences; ***: denotes significance at 10, 5 and 1% respectively.

4.2. Co-integration test

We used the Johansen-Juselius test for determining the long run relationship between the variables in bivariate models (each model represents an equation of Wagner's law), table 3 illustrates the results of trace statistic value, which is greater than the critical value (5% significance level) in just three equations (Peacock-Wiseman; Goffman, Gupta and Musgrave equations), meaning that there is a long-run relationship between real GDP and real government expenditure; real GDP per capita and real government expenditure; real GDP per capita and real government expenditure per capita and real GDP per capita and the ration of the government expenditure to the real GDP, thus the relationship is taken by two variables respectively, and with one vector of co-integration, whereas, these variables have the same behaviour at the long-run term and they move together.

Table 3. Co-integration test results

Null hypotheses	Alternative hypotheses	Eigenvalue	Trace statistic	Critical value 5%	Prob.
Equation 1: RGE = f(RGDP) Peacock-Wiseman (1961)					
r = 0	r = 1	0.1685	14.2972	15.4947	0.0751
r ≤ 1	r = 2	0.1184	5.8004	3.8414	0.0160**

Equation 2: $RGCE = f(RGDP)$ Pryor (1968)					
$r = 0$	$r = 1$	0.1531	9.7556	15.4947	0.3000
$r \leq 1$	$r = 2$	0.0448	2.1090	3.8414	0.1464
Equation 3: $RGE = f(RGDP/N)$ Goffman (1968)					
$r = 0$	$r = 1$	0.2959	18.8709	15.4947	0.0149**
$r \leq 1$	$r = 2$	0.0576	2.7296	3.8414	0.0985
Equation 4: $RGE/N = f(RGDP/N)$ Gupta (1967). Michas (1975)					
$r = 0$	$r = 1$	0.2851	16.7659	15.4947	0.0320**
$r \leq 1$	$r = 2$	0.0283	1.3244	3.8414	0.2498
Equation 5: $RGE/RGDP = f(RGDP/N)$ Musgrave (1969)					
$r = 0$	$r = 1$	0.1686	14.2972	15.4947	0.0751
$r \leq 1$	$r = 2$	0.1184	5.8004	3.8414	0.0160**
Equation 6: $RGE/RGDP = f(RGDP)$ Mann (1980)					
$r = 0$	$r = 1$	0.1396	8.9737	15.4947	0.3678
$r \leq 1$	$r = 2$	0.0436	2.0542	3.8414	0.1518
r: denotes the co-integration vector numbers; **, denotes significance at 10 and 5% significance level.					

4.2. Frequency domain spectral causality test

The final step in this paper is to examine the causal relationship among the variables two by two for each equation to get a clear idea about which of the hypotheses is held in the case of Algerian economy. The results are presented in table 4 and appendix (figures 12 to 17), which indicated that in short run term the neutrality hypotheses is the dominant hypotheses under the six equations, representing the absence of any causal relationship between the two variables, in other words, the mutual effect between government expenditure and economic growth in Algeria over the period of the study is not immediate, and the effect if exist requires a lot of time to reach each of the two variables. However, the causal relationship in the long run term produced different results, the most realized hypotheses is the Wagner's law hypotheses in three equations, indicating the existence of an unidirectional causal relationship between government expenditure and economic growth, when they are taken as real GDP for economic growth and as real government expenditure, real government consumption expenditure and the ratio of government expenditure to GDP. The Keynes view hypotheses with the Goffman equation, demonstrating the unidirectional causal relationship running from government expenditure to economic growth, when the economic growth is measured by the real GDP per capita. Besides to two neutrality hypotheses under Gupta and Michas equation and Musgrave equation show that there is an absence of any causal relationship between the two variables, when we deal with the population measures (government expenditure per capita and GDP per capita).

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Table 4. Causality test results

Equation	Short-run term	Long-run term
Peacock-Wiseman equation	Neutrality hypotheses	Wagner's law hypotheses
Pryor equation	Neutrality hypotheses	Wagner's law hypotheses
Goffman equation	Neutrality hypotheses	Keynes Hypotheses
Gupta and Michas equation	Neutrality hypotheses	Neutrality hypotheses
Musgrave equation	Neutrality hypotheses	Neutrality hypotheses
Mann equation	Neutrality hypotheses	Wagner's law hypotheses

5. Conclusions

The main object of this paper is to investigate the causal relationship between government expenditure and economic growth in Algeria over the period 1970-2017 using various econometric procedures, the recursive unit root tests presented by Phillips et al. (2011 and 2013) to test the explosive behaviors in the series, the co-integration relationship to test the long run relationship among the variables and the frequency causal spectral causality depending on Breitung-Candelon (2006) procedure to examine the existence and the direction of the causality between all the variables. One of the most important contributions of this study is the use of all six equations of Wagner's law to make a comparison between them. Also, this analysis shows several findings according to each econometric study. We firstly found that both of the two variables have some important bubbles (explosive behaviors) over different period, especially after the black decade (1992-1999) but not with the same periods. Then, the Johansen co-integration test indicates the existence of the long run relationship among real GDP and real government expenditure; real GDP per capita and real government expenditure; real GDP per capita and real government expenditure per capita and real GDP per capita and the ration of the government expenditure to the real GDP with two by two relationship respectively with one vector of co-integration. And finally, we established with the causal relationship examination which reveals some contradictory results in the long run term, but the dominant hypotheses is the Wagner's law hypotheses by a unidirectional causal relationship running from economic growth to government expenditure, and also, there is no evidence of any causal relationship between the two variables in the short run term.

REFERENCES

- [1] **Abdullah, H. &Maamor, S. (2010),***Relationship between National Product and Malaysian Government Development Expenditure: Wagner's Law Validity Application. International Journal of Business and Management;*

- [2] **Abizadeh, S. &Yousefi, M. (1998),** *An Empirical Analysis of South Korea's Economic Development and Public Expenditures Growth. The Journal of Socio-Economics*, 27(6), 687-700;
- [3] **Abu-Bader, S. & Abu-Qarn, A. S. (2003),** *Government Expenditures, Military Spending and Economic Growth: Causality Evidence from Egypt, Israel and Syria. Journal of Policy Modeling*, 25(6-7), 567-583;
- [4] **Al-Faris, A. F. (2002),** *Public Expenditure and Economic Growth in the Gulf Cooperation Council Countries. Applied economics*, 34(9), 1187-1193;
- [5] **Burney, N. A. (2002),** *Wagner's Hypothesis: Evidence from Kuwait Using Co-integration Tests. Applied Economics*, 34(1), 49-57;
- [6] **Chipaumire, G., Ngirande, H.&Ruswa, Y. (2014),** *The Impact of Government Spending on Economic Growth: Case South Africa. Mediterranean Journal of Social Sciences*, 5(1), 109;
- [7] **Dritsakis, N. &Adamopoulos, A. (2004),** *A Causal Relationship between Government Spending and Economic Development: An Empirical Examination of the Greek Economy. Applied Economics*, 36(5), 457-464;
- [8] **Ebaid, A.&Bahari, Z.(2019),** *The Nexus between Government Expenditure and Economic Growth: Evidence of the Wagner's Law in Kuwait. Review of Middle East Economics and Finance*, 1-9;
- [9] **Fritsche, U.&Pierdzioch, C. (2016),** *Animal Spirits, the Stock Market, and the Unemployment Rate: Some Evidence for German Data (No. 1/2016). DEP (Socioeconomics) Discussion Papers, Macroeconomics and Finance Series*;
- [10] **Halicioğlu, F. (2003),** *Testing Wagner's Law for Turkey, 1960-2000. Review of Middle East Economics and Finance*, 1(2), 129-140;
- [11] **Homm, U. &Breitung, J. (2012),** *Testing for Speculative Bubbles in Stock Markets: A Comparison of Alternative Methods. Journal of Financial Econometrics*, 10(1), 198-231;
- [12] **Islam, A. M. (2001),** *Wagner's Law Revisited: Co-integration and Exogeneity Tests for the USA. Applied Economics Letters*, 8(8), 509-515;
- [13] **Kalam, M. A. & Aziz, N. (2009),** *Growth of Government Expenditure in Bangladesh: An Empirical Enquiry into the Validity of Wagner's Law. Global Economy Journal*, 9(2), 1850162;
- [14] **Liu, L. C. H., Hsu, C. E. &Younis, M. Z. (2008),** *The Association between Government Expenditure and Economic Growth: Granger Causality Test of US Data, 1947-2002. Journal of Public Budgeting, Accounting & Financial Management*, 20(4), 439-452;
- [15] **Loizides, J. &Vamvoukas, G. (2005),** *Government Expenditure and Economic Growth: Evidence from Trivariate Causality Testing. Journal of Applied Economics*, 8(1), 125-152;
- [16] **Ono, H. (2014),** *The Government Expenditure–Economic Growth Relation in Japan: An Analysis by Using the ARDL Test for Threshold Co-Integration. Applied Economics*, 46(28), 3523-3531;

- [17] Pavia, D. L., Lampman, G. M., Kriz, G. S. & Vyvyan, J. A. (2008), *Introduction to Spectroscopy*. Cengage Learning;
- [18] Phillips, P. C., Shi, S. & Yu, J. (2015), *Testing for Multiple Bubbles: Limit Theory of Real-Time Detectors*. *International Economic Review*, 56(4), 1079-1134;
- [19] Phillips, P. C., Wu, Y. & Yu, J. (2011), *Explosive Behavior in the 1990s Nasdaq: When Did Exuberance Escalate Asset Values?*. *International economic review*, 52(1), 201-226;
- [20] Ram, R. (1986), *Government Size and Economic Growth: A New Framework and Some Evidence from Cross-Section and Time-Series Data*. *The American Economic Review*, 76(1), 191-203;
- [21] Salih, M. A. R. (2012), *The Relationship between Economic Growth and Government Expenditure: Evidence from Sudan*. *International Business Research*, 5(8), 40;
- [22] Shahbaz, M., Khan, S. & Tahir, M. I. (2013), *The Dynamic Links between Energy Consumption, Economic Growth, Financial Development and Trade in China: Fresh Evidence from Multivariate Framework Analysis*. *Energy economics*, 40, 8-21;
- [23] Singh, B. & Sahni, B. S. (1984), *Causality between Public Expenditure and National Income*. *The Review of economics and Statistics*, 630-644;
- [24] Verma, S. & Arora, R. (2010), *Does the Indian Economy Support Wagner's Law? An Econometric Analysis*. *Eurasian Journal of Business and Economics*, 3(5), 77-91;
- [25] Virtanen, T., Tölö, E., Virén, M. & Taipalus, K. (2018), *Can Bubble Theory Foresee Banking Crises?*. *Journal of Financial Stability*, 36, 66-81.

APPENDIX

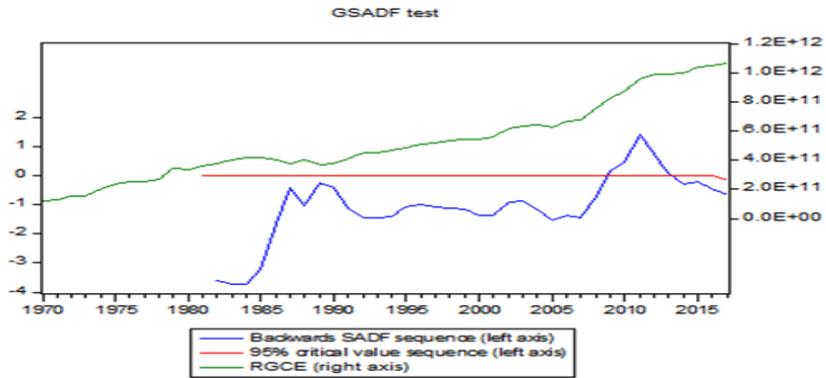


Figure 8.Date-stamping bubble periods in RGCE (GSADF TEST)

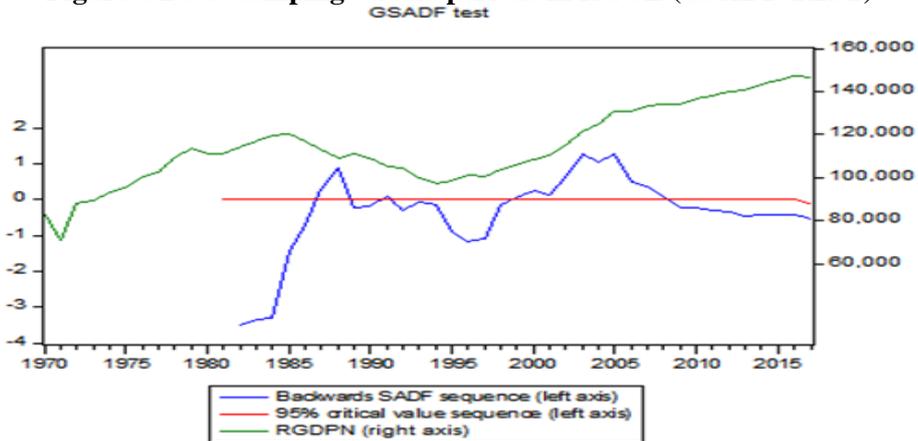


Figure 9.Date-stamping bubble periods in RGDP/N (GSADF TEST)

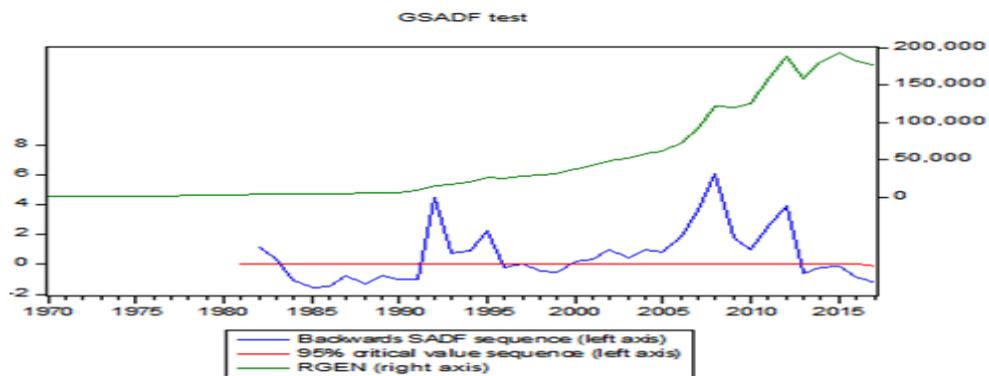


Figure 10.Date-stamping bubble periods in RGE/N (GSADF TEST)

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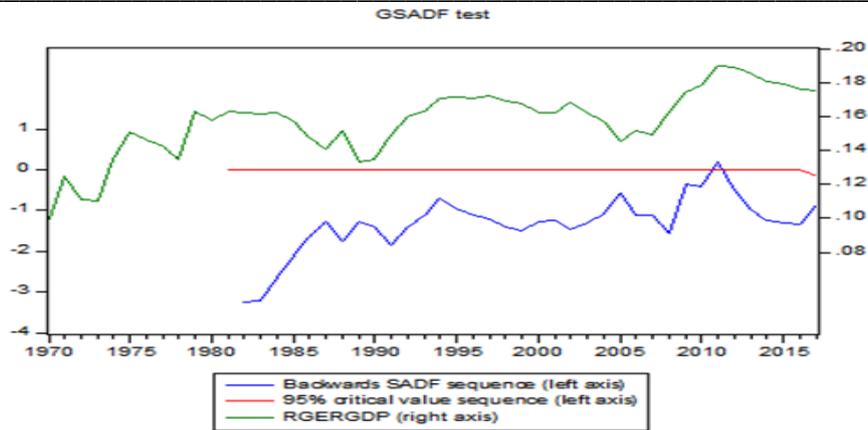


Figure 11. Date-stamping bubble periods in RGE/RGDP (GSADF TEST)

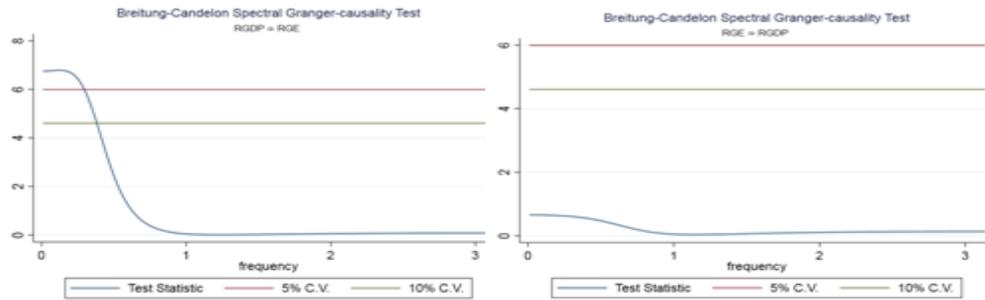


Figure 12. Causality under Peacock-Wiseman equation

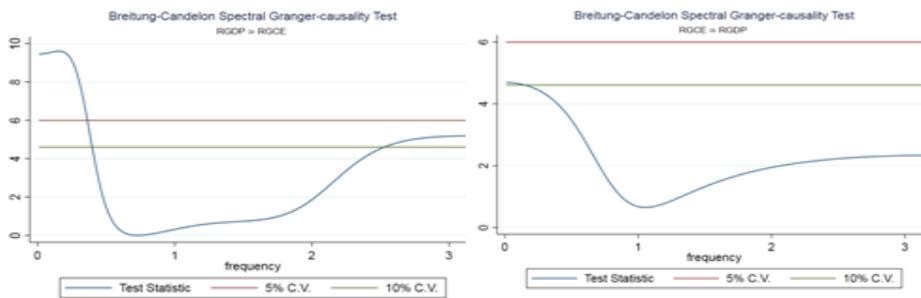


Figure 13. Causality under Pryor equation

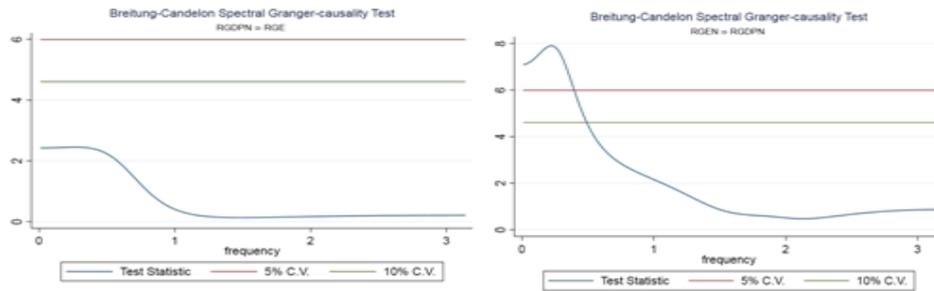


Figure 14. Causality under Goffman equation

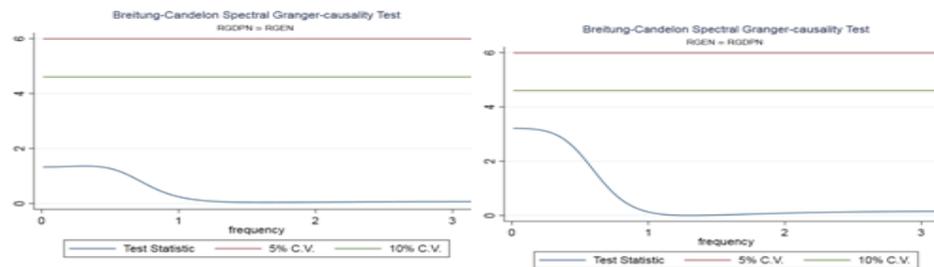


Figure 15. Causality under Gupta and Michas equation

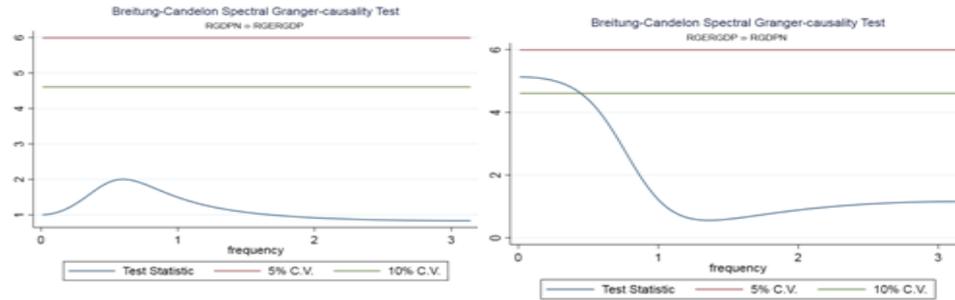


Figure 16. Causality under Musgrave equation

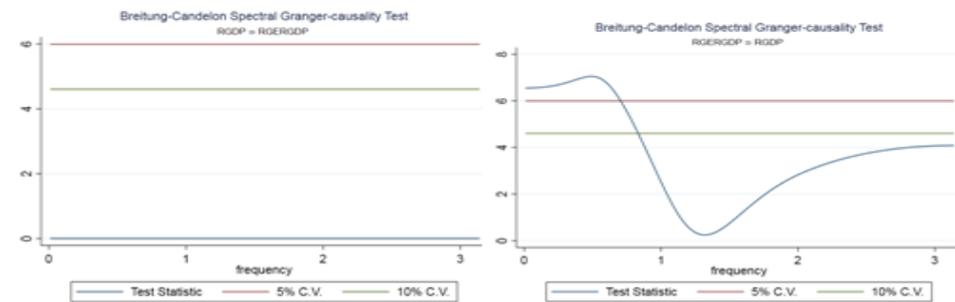


Figure 17. Causality under Mann equation